Brian Conrad was awarded a Barry Prize for Distinguished Intellectual Achievement in 2024. In this video, Sergiu Klainerman of Princeton University interviews Dr. Conrad about his work as a mathematics scholar, why he got involved in fighting efforts to radically lower math standards in high-school curricula and admissions requirements, and why everyone should know something about math even in a world where computers are widely available.

Sergiu Klainerman, Princeton University

Brian Conrad, congratulations for receiving the Barry Prize. I read on the Barry Prize citation that you have done pioneering work in number theory, most notably in connection with the Taniyama-Shimura-Weil conjecture. You have also been an important voice for maintaining rigor in mathematics education, particularly so in the recent debates concerning California's mathematical framework. Let's start with the first part of the citation.

Brian Conrad, Stanford University

I'll try to give a rough sense of a couple of things that I did. I should take a step back. In number theory, we're generally trying to understand a whole number or fractional solutions to equations, usually equations in 2 or more variables are quite different from what you see in high school. One tool that often gets used is divisibility. A classic example would be the Legendre 3-square theorem, which describes which whole numbers are a sum of 3 perfect squares. For example, 59 is 49 plus 9 plus 1, whereas 63 is not a sum of 3 perfect squares. This theorem says an odd whole number is a sum of 3 squares as long as it does not leave a remainder of 7 when you try to divide it by 8.

There are many situations in number theory when we try to analyze whether an equation in 2 or more unknowns has a whole number of fractional solutions. It's very useful to analyze these questions by focusing on how they relate to divisibility properties of the variables involved or some of the quantities in the equation. We refer to this as *modular arithmetic*. We may be studying remainders when you divide by 8 or remainders when you divide by some other prime. When we take a question about whole numbers and turn it into this world of modular arithmetic, it takes the problem involving infinitely many integers into a very finite number system. So, it turns out to be a very tractable way to find obstacles to whole number solutions to equations.

This idea of modular arithmetic was actually initiated by Gauss when he wrote the famous book that spawned modern number theory, *Disquisitiones Arithmeticae*. We found in analyzing questions in number theory that if we take the original problem and recast it in terms of this world of modular arithmetic, it can give us obstructions to the solvability, but it can also sometimes give us criteria that may be sufficient for the existence of the solutions that we're looking for. Gauss studied this in a certain realm of cases, but a situation that number theorists have studied a lot in the last 50 years is a special class of 2-variable equations called *elliptic curves*. Elliptic curves are roughly equations of the form y squared equals a cubic polynomial in x, like y squared equals x cubed minus 2, or something like that. These may sound quite special, but they turn out to have a lot of interesting structure and to be a very good testing ground for many general conjectures in number theory. When we try to study these special cubic equations called elliptic curves, even though they're not ellipses, it turns out that once again, if we want to understand the totality of the whole number or fractional solutions to these equations, points on these curves with whole number coordinates, we can introduce considerations in modular arithmetic. We can study both sides of the equation when we look at remainders, when we try to divide by different primes. When we try to assemble this modular arithmetic information, we get an infinite sequence of numbers, and we assemble those into a single function called the *elliptic curve L-function*. It's like in calculus, we may have a power series that assembles the implementing coefficients into a single function. Starting from an elliptic curve equation, we can introduce its working modulo prime, study the divisibility properties of the equation when we look at remainders, modulo, all the different primes, and we can assemble all that modular arithmetic information into the single elliptic curve L-function.

There are many conjectures about these L-functions, but the most basic thing one needs to establish to study the more difficult conjectures, many of which are wide open, is a property that we call *entirety* or *entireness*. This is a special property that people didn't know how to solve, to establish, for a long time. When Wiles solved Fermat's Last Theorem, the key was he came up with a novel criterion to justify that the L-function associated with an elliptic curve has this special property of entireness. He was able to establish this for enough elliptic curves to solve Fermat's Last Theorem, but the question remained open for all elliptic curves, and Richard Taylor, Christophe Breuil, Fred Diamond, and I extended Wiles's method to handle all elliptic curves. I should say, earlier this year, these same techniques that Wiles initiated have now been extended to some 2-dimensional geometric objects, certain kinds of special surfaces. So, these techniques that have been developed keep being generalized and extended to more and more situations. It's become quite a powerful technique within number theory.

I'll just briefly mention something I did in more recent years, again connecting back to this modular arithmetic idea. There's a very powerful analogy that we call the *Rosetta Stone*, going back to a letter that André Weil wrote to his philosopher sister Simone Weil in the 1940s, that in number theory we want to study questions about whole numbers and fractions, but there are very interesting analogies between numbers and polynomials, such as a prime number may correspond to an irreducible polynomial, or a fraction like five-thirds, may be somewhat analogous to a rational function. In any event, under this analogy between numbers and polynomials, we have a bridge. We may regard it as a Rosetta Stone; there are certain concepts and results on one side, and we try to formulate the analog on the other side. Sometimes on this other side, related to polynomials, we may have more geometric insight into what's going on, and that can suggest useful ideas.

In any event, something we try to do a lot in number theory is to find more and more instances of this Rosetta Stone, so we can take a problem we don't know how to study on one side, translate it to the other side, and make progress, or even just develop the corresponding analogy on the other side to give us more confidence in the value of this Rosetta Stone that we use to inspire algebraic ideas coming from geometry. The reason I bring this up is, in the past 10 years, Gopal Prasad, Ofra Gaber, and I developed a new theory of continuous symmetry on the function field side of the Rosetta Stone, what we call *pseudo-reductive groups*. These are analogs on one side of this number theory dictionary to the very highly developed theory of what are called *arithmetic Lie groups*, which have been used a lot for studying systems of equations that have a lot of symmetry in the context of ordinary numbers. We developed a kind of parallel theory of continuous symmetry on the other side of this Rosetta Stone, and it led to a variety of nice finiteness theorems and other results that are parallel to what was known on the other side. That has led to a variety of other developments in this study of symmetry in a somewhat nonclassical setting.

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Klainerman

At least I could understand a little bit of what you are saying, so that was an excellent explanation. Connected with this Rosetta Stone, I saw that a Breakthrough Prize was announced today for the Langlands program.

Conrad

Within number theory, the Langlands program tries to establish, again, a kind of bridge between very different-looking kinds of structures and to take problems in one setting and transform them into some other one where there's more structure to grab on to. In the Langlands program is one side, you might say the more classical side, where there has been some progress, Wiles's work and others, but many things are unknown. On this other side of the Rosetta Stone, which is much closer to geometry. . . .

Klainerman

It's easier, I presume.

Conrad

I would say it's different. You're absolutely correct that the availability of additional geometric tools definitely makes certain concepts more tractable than they are on the more classical side.

On the other hand, what's more important is it can motivate what type of geometric structure to then search for on the classical side. An instance of this in the last several years is the work of Peter Scholze and Laurent Fargues on what's called the *geometrized local Langlands program*. They took ideas inspired by this geometric side, and they figured out how to build the right kinds of structures in the context of what's called the *classical local Langlands correspondence*. It's led to tremendous advances there. But people never would have known what to look for if they didn't have the experience on this other side. So, you're correct that the availability of extra geometric tools does sometimes lead to greater progress on this side. But nonetheless, it can also inspire the search, what kinds of concepts and structures to look for. It has been more than just an analogy, it's actually inspired people to find the right kinds of ideas on the classical side as well, which has led to some advances there too.

Klainerman

That's fascinating. I really appreciate this myself, and I'm sure that the people who will listen to this would also enjoy it.

Let's go to the second part of your Barry Prize, which concerns the California mathematical framework. I know that you became very involved and had a very positive impact on the recent debates to do with it. I'd like to know what the situation is. How did you get involved? And what is the situation today?

Conrad

My father has been a high school math teacher for all of his career. I probably had somewhat more awareness about things happening in the high schools, maybe more than many professional mathematicians. In addition to being a professor in my department, I've been the director of undergraduate studies for math at Stanford for roughly the past 10-plus years. In that role, I've spoken

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with colleagues across many other quantitative fields, statistics, computer science, biology, and so on. So, I had some awareness of the ways in which the math topics that many students learn in this department play into what they need for their work in other subjects, and correspondingly, the sorts of tools students are learning in high school that still get used in many of these other fields.

In 2021, I began reading in the news about proposals to essentially remove large chunks from the conventional math curriculum under the claim that this would allow replacing obsolete Sputnik-era topics with more contemporary things such as data science. A lot of buzz words were being used, which could have superficially sounded very plausible in the sense that if parents don't know where the material in the curriculum goes after high school, it could make sense that a lot of those math topics have become obsolete. But you know and I know that actually a lot of the core material in the conventional high school curriculum is absolutely foundational for modern science, and frankly, even modern data science. Even though calculus was developed for physics, nowadays, calculus is used for optimization, trigonometry is used in video games, all these recommender systems, AI—all of these things rely crucially on a large part of the usual algebra and calculus and such material that's taught in the high schools and early courses in college.

Seeing these claims being made in the media that I recognized as being quite misleading, I decided that because of my position as the director of undergraduate studies at this particular university, if I would speak up, I could potentially have a positive impact on how the public debate would go around these topics and hopefully have a useful effect on making improvements to the proposals that were being made. I sat down and read the entirety of the proposed 1,000-page framework document and documented the concerns and also suggestions for how to improve things. I worked, of course, together with some other people. In the end, I feel that what happened in California did avert what could have been the problematic development of gutting the university admission standards for what kids need to know for a solid four-year quantitative degree, but it also averted the problem of potentially having the high school curriculum replace substantive math content with fluff.

One very useful thing that came out of the entire discussion was to bring to wider public awareness the fact that, indeed, we do need to provide kids with a sense of how what they're being taught in school does pertain to contemporary things, that we're not just teaching them old material for the sake of learning it for itself, but rather for them to see that these topics that you and I recognize as central to mathematics really do remain the core for what people need to get a meaningful college degree in many quantitative fields, whether it's statistics, computer science, AI, data science, and so on. We did have a partially positive effect in California in the sense that many of the problematic proposals were eventually toned down a lot. Although in the end, the California math framework didn’t necessarily achieve as much as it could have, a lot of the big concerns were largely addressed in the final version.

On the other hand, these things moved on to other states. For example, after the attempts to water down the high school curriculum in California did not pan out, in Connecticut, a similar thing arose where there was an attempt to try to get the public university, the University of Connecticut, to water down the math requirements for college admissions. That was, again, presented under a kind of argument of making things more accessible, but in fact, watering down the math requirements for college admissions would really have the effect of making quantitative degrees out of reach for the students who didn’t learn what they should have in high school.

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In any event, many parents got involved in Connecticut. I spoke to people there, provided them with information, and they were able to stop the proposal there from getting very far. Fortunately, in Connecticut, things are now in a reasonable state. Things have been occurring in other states, and I think the main takeaway lesson from this is that there's a lot of value that can be had by not just professional mathematicians, but also by people in other quantitative fields, computer science, statistics, also business and industry, getting involved in these discussions to enable both parents and school districts and the people who set the curriculum to appreciate ways in which the content of the curriculum really does still remain as relevant now as it did long ago.

Klainerman

So, in the end, you are saying that you were able to influence the debate and you made corrections, but you are not entirely satisfied. Why?

Conrad

There's certainly ample room for improving the material that the teachers get in the high schools with which they can help kids to see the value of what they're teaching in the classrooms, for example, enabling kids to see trigonometry getting used in video games or maybe connecting some inequality ideas to probability and other things. If the process by which the textbook approvals had incorporated a requirement that the publishers provide more indications of contemporary relevance, that would be very useful to the teachers when they're trying to help the kids to get more motivated to learn the material in the curriculum. Unfortunately, that was not able to be included.

There is a lot of room for helping teachers to see the contemporary relevance so that they can convey that to their students. Because of course, even when you or I learn about a piece of advanced mathematics, we're also driven by motivation. We see its value to do something that we find interesting. Quite reasonably, many kids or their parents in the high schools can feel similarly about why they are learning certain topics. If publishers could do more to help teachers to show kids these examples of contemporary relevance, it could help a lot to motivate them to persist in getting through the material that they may find initially quite challenging in high school.

Klainerman

I certainly sympathize with everything you are saying. What do you perceive as being the main challenges that you are facing now? Are you interested in other issues that accompany education?

Conrad

On the education side, the main challenge is the way that math is used tends to be under the hood. So, it can be very easy for people to fall into the mistaken belief that because of the ubiquity of computers people don't need to understand some of these difficult mathematical ideas. We would never apply that principle to writing just because there are spell checkers and because AI can write essays, therefore, people don't need to know how to write anymore. Nobody would make that argument. But an argument roughly of that nature is being made about math in various places. To be fair, some of the people making these arguments may just be genuinely misinformed.

One of the big challenges, then, is to enable more people to open up the hood and look underneath.

Klainerman

When it comes to the extraordinary success of AI, of large language models, it’s completely based on optimization.

Conrad

Exactly. To people who don't know how it works, it can all seem like black magic. Of course, the people who are going to push the development of these things and use them wisely will have some awareness about what's going on under the hood. There is a lot of potential for enabling more students to see the utility of these things. But somebody has to give the teachers better material with which they can convey that. That's still a huge problem.

Traditionally, this has been one of the difficulties with appreciating the role of mathematics in wider society. With physics and chemistry and medicine and biology, people can see more directly how the scientific advances pertain to what people use every day. You and I recognize that there's all this math lying underneath, it's lurking inside the computer. The creation of more material to help people to see that that mathematical knowledge enables other people to develop things further would go a long way toward improving the reputation of mathematics but also motivating more kids to persist through learning some of these more difficult topics in high school.

Klainerman

It's pretty clear that the range of mathematical applications has increased dramatically.

Conrad

But, as I say, it tends to be under the hood, out of view. If we could enable more people to see where it's lurking there and give kids some activities or something to enable them to make direct contact between what they're learning and those applications, even in a toy model, it probably would help a lot. There are organizations working on that sort of thing. There's an organization in Texas called Skew the Script, which tries to create such material.

Klainerman

Well, thanks a lot. This was extremely useful. I wish you continued success both as a mathematician and as an intellectual involved in various issues, most important, mathematical education, of course.

Conrad

Thank you.

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